tip geometry, and the applicability to other unsteady flows remain for future investigation.

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Airplane Flight Through Wind-Shear Turbulence

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Nomenclature

a,b	= time-dependent parameters defining anisotropic effects on two-point velocity correlation structure
A,B,C,D	= arbitrary functions in the invariant algebraic representation of correlation tensor(s)
$C_{ij}(r)$	= two-point velocity correlation tensor for anisotropic turbulence
f,g	= longitudinal, transverse correlation functions for isotropic turbulence
$K(t)$	= variable defined in Eq. (3); $K(t)>0$ for all t
L	= transverse correlation length for anisotropic turbulent downwash
M	= slope of mean-flow shear
r	= magnitude of $\boldsymbol{r}$, i.e., $r=\lvert\boldsymbol{r}\rvert$
$\boldsymbol{r}$	= separation vector for turbulence velocity correlations
$R_{ij}(r)$	= two-point velocity correlation tensor for isotropic turbulence
$S_{ili}(r)$	= two-point "turbulence self-interaction" tensor
$\boldsymbol{u}$	= turbulence velocity $=(u,v,w)$
$\bar{U}$	= mean-flow velocity $=(\bar{U},0,0)$
α	= glide-slope angle for aircraft landing (or takeoff)
λ	= unit vector defining preferred direction of turbulence anisotropy
Λ	= longitudinal correlation length for isotropic turbulence
$\Delta\Lambda$	= change in correlation length due to turbulence anisotropy
ξ	= scalar variable defined for anisotropic turbulence, $=\lambda\cdot\boldsymbol{r}$
σ	= turbulence intensity

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Introduction

IN a recent Note[1] it was established why traditional schemes (see Ref. 2 and references cited therein) for simulating air turbulence typically lead to "unacceptable" results. In particular, it was shown that the historical Gaussian time-history approach as well as the modern modified Gaussian approach do not at all model the nonlinear convective effect which is eminently characteristic of hydrodynamic turbulence. In fact, both approaches completely disdain this phenomenon and proceed as though the turbulence and mean-flow velocities, u and $\bar{U}$, respectively, can be linearly superimposed with no related interplay. Note that convection is analogous to skewness in the probability density function (pdf) of the turbulence, and any pdf that is symmetric about $u\equiv 0$ automatically has zero skewness.

In this Note the effect of *mean-flow shear* ("wind shear") on the turbulence structure is examined. Shear is the description given to a mean flow, which is spatially nonconstant, say $\bar{U}\sim\bar{U}(x)$, and its most pronounced effect is the strong anisotropy it unfortunately generates. Shear is encountered by an airfract in the atmospheric boundary-layer when the aircraft is usually in the takeoff or landing mode. It may also be encountered at high altitudes; however, these encounters, from the viewpoint of aviation safety, are potentially not as hazardous. In the sequel it is shown that as an airplane encounters a "microburst" even a small amount of anisotropy produces a significant time variation in the sensed magnitude of the *integral scale*—a measure of the degree of "randomness' in turbulence; "large" integral scale ~ moderately random turbulence while "small" integral scale ~ highly random turbulence.

Anisotropy for Flight Simulation

The two-point velocity correlation for anisotropic turbulence is[3]

$$\langle u_i(\boldsymbol{x})u_j(\boldsymbol{x}+\boldsymbol{r})\rangle = C_{ij}(\boldsymbol{r}) = Ar_ir_j + B\delta_{ij} + C\lambda_i\lambda_j + D(\lambda_ir_j + r_i\lambda_j)$$

where A, B, C, and D are arbitrary functions of $r^2=\boldsymbol{r}\cdot\boldsymbol{r}$ and $\xi=\lambda\cdot\boldsymbol{r}=r\cos\theta$ (also t, but this dependence is suppressed here for convenience). It can be readily shown[3] that A, B, and C are even functions of both r and ξ, but D, on the other hand, is an odd function of ξ. For boundary-layer turbulence, the unit vector λ can be assumed to have components $\lambda=(0,0,1)$, indicating that the vertical (z) direction is the proverbial "preferred direction of the flow." Note that this expression for $C_{ij}(\boldsymbol{r})$ is different from that for *isotropic* turbulence, namely, $R_{ij}(\boldsymbol{r})=Ar_ir_j+B\delta_{ij}$, where $A\sim A(r \text{ only})=\sigma^2(f-g)/r^2$ and $B\approx B(r \text{ only})=\sigma^2 g$; consistent with isotropic theory, $f\sim$ *longitudinal* correlation function, $g\sim$ *transverse* correlation, and $\sigma\sim$ *intensity*. When an airplane is either taking off or landing, its flight path is generally such that $\xi=\lambda\cdot\boldsymbol{r}=r\sin\alpha\simeq r\alpha$ (see Fig. 1); therefore, for the considered simulation purpose, the preceding functions A, B, C, and D can be roughly approximated as $A(r^2,\xi)\simeq A(r^2,0)$, $B(r^2,\xi)\simeq B(r^2,0)$, $C(r^2,\xi)\simeq C(r^2,0)$, and $D(r^2,\xi)\simeq D(r^2,0)\equiv 0$. Accordingly, the corresponding turbulence correlations are

$$C_{11}(\boldsymbol{r}) = r^2A + B \simeq \sigma^2 f(r)$$

$$C_{22}(\boldsymbol{r}) = B \simeq \sigma^2 g(r)$$

$$C_{33}(\boldsymbol{r}) = \alpha^2r^2A + B + C \simeq B + C \neq \sigma^2 g(r)$$

$$C_{13}(\boldsymbol{r}) = \alpha r^2 A = \alpha\sigma^2\{f(r)-g(r)\}$$

where $\boldsymbol{r}=(r_x,0,r_z)\simeq(r,0,\alpha r)$; note that A and B here are identical to the isotropic case, namely, $A=\sigma^2(f-g)/r^2$ and

$B=\sigma^2 g$. The related expression for $C(r^2,0)$, however, is determined from the equation [cf., Ref. 3, Eqs. (2.14) and (2.15)], $\alpha(\partial C/\partial r)\simeq 0$; this equation requires that $C(\cdots)$ be of the form $C(r^2,0)\simeq a|r|+b$, where a and b are time-dependent parameters and $|a|\sim \mathcal{O}(\alpha)$. The specific anisotropic correlations for flight simulation are therefore

$$\langle u(x)u(x+r)\rangle \simeq \sigma^2 f$$

$$\langle v(x)v(x+r)\rangle \simeq \sigma^2 g$$

$$\langle w(x)w(x+r)\rangle \simeq \sigma^2 g + a|r| + b$$

$$\langle u(x)w(x+r)\rangle \simeq \alpha\sigma^2 (f-g)$$

Dynamical Implications

The dynamical implications of the proposed anisotropy can be determined by writing the Navier-Stokes equation for $C_{ii}(r,t)=r^2A+3B+C\simeq\sigma^2(f+2g)+C$, and then subtracting from it the corresponding equation for $R_{ii}(r,t)=r^2A+3B\equiv\sigma^2(f+2g)$. The necessary result is (cf., Ref. 4).

$$\frac{\partial C}{\partial t}+2M\alpha\sigma^2(f-g)+M\alpha r\sigma^2\frac{\partial(f+2g)}{\partial r}\simeq 0 \tag{1}$$

In obtaining this result it has been assumed that the "turbulence self-interaction" term $S_{ili}(r,t)=\langle u_i(x,t)u_l(x,t)u_i(x+r,t)\rangle$ is negligible when compared to the "mean-flow interaction" term, $(\partial U_i/\partial x_l)C_{li}(r,t)$; simultaneously, $\partial^2 C/\partial r^2\equiv 0$, while the mean flow is defined as $\bar{U}=\{\bar{U}(z),0,0\}=(Mz,0,0)$, $M\equiv$const. Since $g=f+rf'/2$ (Ref. 5), it quickly follows that $(f-g)=-rf'/2$ and $(f+2g)=3f+rf'$; $f'\sim\partial f/\partial r$. Equation (1) then integrates in r into

$$\frac{\mathrm{d}}{\mathrm{d}t}(\sigma^2\Delta\Lambda)-M\alpha\sigma^2\Lambda\simeq 0 \tag{2}$$

where

$$\int_0^\infty C(r)\mathrm{d}r\sim\sigma^2\Delta\Lambda \text{ and } \Lambda=\int_0^\infty f\mathrm{d}r$$

the solution of Eq. (2) is

$$\sigma^2\Delta\Lambda\simeq M\alpha K(t) \tag{3}$$

where $K(t)=\int\sigma^2\Lambda\mathrm{d}t\sim$ a time-dependent parameter that is always *positive*, i.e., $K(t)>0$ for all t since σ and Λ are always positive by definition. From the corresponding ("low" Reynolds number) isotropic theory,[5] $\sigma^2\Lambda\sim t^{0.8n}$ and, therefore,

$$K(t)\sim t^{1+0.8n} \tag{4}$$

For the power-law form of σ^2, namely, $\sigma^2\sim t^{-n}$, it follows that

$$\Delta\Lambda\sim M\alpha t^{1+1.8n} \tag{5}$$

and, for $n=1$,

$$\Delta\Lambda\sim M\alpha t^{2.8} \tag{6}$$

Application to Flight Through a Microburst

In landing through a microburst (see Fig. 2) the turbulence component of interest is invariably $\langle w(x_1,z_1)\ w(x_2,z_2)\rangle\simeq C_{33}(r,z)$, where $x_2-x_1\simeq r$ and $z=(z_1+z_2)/2$. Here the turbulence is assumed to be "frozen," i.e., independent of t, but necessarily *nonhomogeneous*; the integral scale, L, for this component is

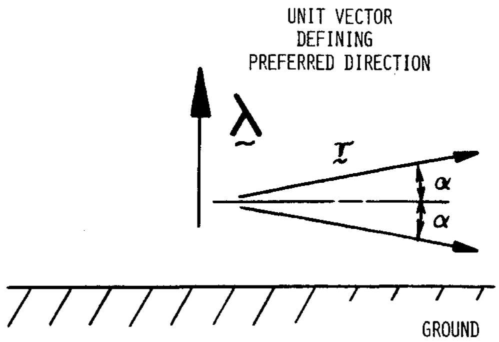

Fig. 1 Glide-slope angle for airplane landing (or takeoff). For $\alpha\sim$"small," $\xi=\lambda\cdot r\simeq r\alpha$.

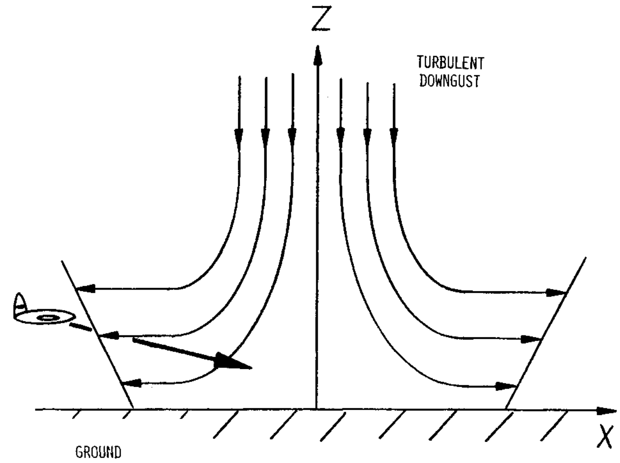

Fig. 2 Aircraft attempting to land through a turbulent microburst. Note that $\{M\alpha\}>0$ on the headwind side, while $\{M\alpha\}<0$ on the tailwind side.

$$L=\sigma^{-2}\int_0^\infty C_{33}(r,z)\mathrm{d}r\sim\int_0^\infty g\mathrm{d}r+\sigma^{-2}\int_0^\infty C(r,z)\mathrm{d}r$$

As the airplane enters the microburst the nonhomogeneous z dependence of the turbulence is sensed as a t dependence by the moving airplane and, in particular,

$$L=\tfrac{1}{2}\{\Lambda+2\sigma^{-2}M\alpha K(t)\}=\Lambda/2+\Delta\Lambda$$

Recall that Λ is the longitudinal integral scale for isotropic turbulence and the effects of shear (anisotropy) are introduced through $\Delta\Lambda=\sigma^{-2}M\alpha K(t)$. Λ itself, though, can be taken here to be approximately constant. As the airplane encounters the microburst, the correlation length L sensed by the aircraft is $L>\Lambda$, since both M and α are negative for this part of the flight path; i.e., the "sensed" turbulence is *less random* than what is predicted by isotropic theory, and the pilot has no trouble handling the aircraft. However, as the airplane continues to move through the microburst and eventually enters the region where $M>0$, the L sensed by the aircraft is $L<\Lambda$ and the sensed turbulence there is *more random* than predicted by isotropy. This condition, together with the resulting loss of lift, creates the hazardous environment for an aircraft as it attempts to land through the microburst.

Concluding Remarks

For faithful flight-simulator application, then, the required changes in integral scale need to be incorporated. These changes stem from the combined values of M and α, so that the stronger the shear the more pronounced is its effect on the "randomness" of the sensed turbulence. The foregoing does not fully model the presence of the boundary, but its neglect is not crucial to the general results of the analysis.

Acknowledgments

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Lower-Side Normal Force Characteristics of Delta Wings at Supersonic Speeds

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Nomenclature

a	= normal force ratio
C_D	= drag coefficient
C_L	= lift coefficient
C_N	= normal force coefficient
C_N^ℓ	= wing lower-side normal force coefficient
K	= shape factor
M	= Mach number in freestream
α	= angle of attack
α_n	= angle of attack normal to wing leading edge
β	$=\sqrt{M^2-1}$
γ	= ratio of specific heats
ϵ	= wing apex half-angle
Λ_{LE}	= wing leading-edge sweep angle
θ	= profile apex half-angle

Subscripts

α	= slope vs angle of attack
ℓ	= lower surface
$(\ )_p$	= attached (potential) flow
nf	= normal to disk front surface

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Introduction

DURING work with an engineering method for estimating characteristics of flat, sharp leading-edge delta wings at supersonic speeds, it became clear that the lower-side normal force contribution could be reasonably well represented by a two-term expression analogous to the long applied estimation of the normal force of slender bodies. Use is made of experimental data from several sources: the lower-side normal force characteristics over the incidence presented by Wood and Miller[1,2]; the lift curve slope results obtained by Lampert[3]; and Hoerner's[4] compilation of data on disks, analytically represented in Ref. 5. For lack of an accurate estimate of the lift curve slope of the lower-side contribution, an empirical assumption had to be made in the work presented here. This weakness of the present result can be removed sooner or later.

Analytical Expressions

Working from the results of Wood and Miller[1,2] concerning the normal force contribution of the lower side of the wing in the restricted interval, $0.5<\beta\tan\epsilon<1$, a possible and simple representation of the partial coefficient is

$$C_N^\ell = (C_{N_\alpha}^\ell)_p \sin\alpha\cos\alpha + C_{D_{nf}}\sin^2\alpha \le C_{D_{nf}} \tag{1}$$

where the first term is the lower side normal force curve slope expressed by

$$(C_{N_\alpha}^\ell)_p = a_\ell C_{N_\alpha} \tag{2}$$

characterizing the potential-flow-like part of the normal force. $C_{N_\alpha} \simeq C_{L_\alpha}$ is obtained from experiments by Lampert.[3] The second term represents the nonlinear contribution to the lower-side normal force. The coefficient is the face drag coefficient of a disk in a flow normal to the disk. The coefficient is compiled by Hoerner[4] from experiments and is represented analytically including a Mach number correction in Ref. 5. The expressions are

$$C_{D_{nf}} = \frac{K(q''/q) - 2(1-K)}{\gamma M^2} \tag{3}$$

where

$$\frac{q''}{q} = 1.84 - \frac{0.76}{M^2} + \frac{0.166}{M^4} + \frac{0.035}{M^6} + \ldots \tag{4}$$

and

$$K = 0.935 - 0.023\left[1 - \left(1 - \frac{1}{M^2}\right)^{1/4} \times \left(\frac{M-3}{0.25+|M-3|}\right)^{3/5}\right], \qquad M > 0.8 \tag{5}$$

Equation (4) is the gasdynamic relation between the stagnation pressures at the face surface and in the freestream.

At high angles of attack, the terms in Eq. (1) will add up to a sum larger than the front face drag of disks at 90 deg. This is not realistic; therefore, C_N^ℓ is limited as shown in Eq. (1). It might be assumed that the overshoot is precisely compensated by a corresponding loss in the potential flow contribution; however, this situation will not occur because only angles of attack well below 45 deg are treated here.

Equations (1) and (2), after inserting $a_\ell = 0.41$ and calculating the angles of attack normal to the leading edge $[\alpha_n = \tan^{-1}(\tan\alpha/\sin\epsilon)]$, give the curves shown in Fig. 1. The Mach number dependency is quite large for the less swept wings and decreases markedly for increasing sweep.